NAME ______________________________________ DATE ____________________________ PERIOD _____________

6-7 Study Guide and Intervention

Solving Radical Equations and Inequalities

Solve Radical Equations The following steps are used in solving equations that have variables in the radicand. Some algebraic procedures may be needed before you use these steps.

Step 1 Isolate the radical on one side of the equation.
Step 2 To eliminate the radical, raise each side of the equation to a power equal to the index of the radical.
Step 3 Solve the resulting equation.
Step 4 Check your solution in the original equation to make sure that you have not obtained any extraneous roots.

Example 1: Solve $2\sqrt{4x+8} - 4 = 8$.

$2\sqrt{4x+8} - 4 = 8$	Original equation
$2\sqrt{4x+8} = 12$	Add 4 to each side.
$\sqrt{4x+8} = 6$	Isolate the radical.
$4x + 8 = 36$	Square each side.
$4x = 28$	Subtract 8 from each side.
$x = 7$	Divide each side by 4.

Check

$2\sqrt{4(7)+8} - 4 \stackrel{?}{=} 8$

$2\sqrt{36} - 4 \stackrel{?}{=} 8$

$2(6) - 4 \stackrel{?}{=} 8$

$8 = 8$

The solution $x = 7$ checks.

Example 2: Solve $\sqrt{3x+1} = \sqrt{5x} - 1$.

$\sqrt{3x+1} = \sqrt{5x} - 1$	Original equation
$3x + 1 = 5x - 2\sqrt{5x} + 1$	Square each side.
$2\sqrt{5x} = 2x$	Simplify.
$\sqrt{5x} = x$	Isolate the radical.
$5x = x^2$	Square each side.
$x^2 - 5x = 0$	Subtract 5*x* from each side.
$x(x - 5) = 0$	Factor.
$x = 0$ or $x = 5$	

Check

$\sqrt{3(0)+1} = 1$, but $\sqrt{5(0)} - 1 = -1$, so 0 is not a solution.

$\sqrt{3(5)+1} = 4$, and $\sqrt{5(5)} - 1 = 4$, so the solution is $x = 5$.

Exercises

Solve each equation.

1. $3 + 2x\sqrt{3} = 5$
$\frac{\sqrt{3}}{3}$

2. $2\sqrt{3x+4} + 1 = 15$
15

3. $8 + \sqrt{x+1} = 2$
no solution

4. $\sqrt{5-x} - 4 = 6$
–95

5. $12 + \sqrt{2x-1} = 4$
no solution

6. $\sqrt{12-x} = 0$
12

7. $\sqrt{21} - \sqrt{5x-4} = 0$
5

8. $10 - \sqrt{2x} = 5$
12.5

9. $\sqrt{4+7x} = \sqrt{7x-9}$
no solution

10. $4\sqrt[3]{2x+11} - 2 = 10$
8

11. $2\sqrt{x-11} = \sqrt{x+4}$
16

12. $(9x-11)^{\frac{1}{2}} = x + 1$
3, 4

6-7 Study Guide and Intervention *(continued)*

Solving Radical Equations and Inequalities

Solve Radical Inequalities A **radical inequality** is an inequality that has a variable in a radicand. Use the following steps to solve radical inequalities.

Step 1 If the index of the root is even, identify the values of the variable for which the radicand is nonnegative. **Step 2** Solve the inequality algebraically. **Step 3** Test values to check your solution.

Example: Solve $5 - \sqrt{20x + 4} \geq -3$.

Since the radicand of a square root must be greater than or equal to zero, first solve $20x + 4 \geq 0$.

$20x + 4 \geq 0$

$20x \geq -4$

$x \geq -\frac{1}{5}$

Now solve $5 - \sqrt{20x + 4} \geq -3$.

$5 - \sqrt{20x + 4} \geq -3$	Original inequality
$\sqrt{20x + 4} \leq 8$	Isolate the radical.
$20x + 4 \leq 64$	Eliminate the radical by squaring each side.
$20x \leq 60$	Subtract 4 from each side.
$x \leq 3$	Divide each side by 20.

It appears that $-\frac{1}{5} \leq x \leq 3$ is the solution. Test some values.

$x = -1$	$x = 0$	$x = 4$
$\sqrt{20(-1) + 4}$ is not a real number, so the inequality is not satisfied.	$5 - \sqrt{20(0) + 4} = 3$, so the inequality is satisfied.	$5 - \sqrt{20(4) + 4} \approx -4.2$, so the inequality is not satisfied.

Therefore, the solution $-\frac{1}{5} \leq x \leq 3$ checks.

Exercises

Solve each inequality.

1. $\sqrt{c - 2} + 4 \geq 7$

$c \geq 11$

2. $3\sqrt{2x - 1} + 6 < 15$

$\frac{1}{2} \leq x < 5$

3. $\sqrt{10x + 9} - 2 > 5$

$x > 4$

4. $8 - \sqrt{3x + 4} \geq 3$

$-\frac{4}{3} \leq x \leq 7$

5. $\sqrt{2x + 8} - 4 > 2$

$x > 14$

6. $9 - \sqrt{6x + 3} \geq 6$

$-\frac{1}{2} \leq x \leq 1$

7. $2\sqrt{5x - 6} - 1 < 5$

$\frac{6}{5} \leq x < 3$

8. $\sqrt{2x + 12} + 4 \geq 12$

$x \geq 26$

9. $\sqrt{2d + 1} + \sqrt{d} \leq 5$

$0 \leq d \leq 4$