

6-7 Study Guide and Intervention

Solving Radical Equations and Inequalities

Solve Radical Equations The following steps are used in solving equations that have variables in the radicand. Some algebraic procedures may be needed before you use these steps.

Step 1 Isolate the radical on one side of the equation.

Step 2 To eliminate the radical, raise each side of the equation to a power equal to the index of the radical.

Step 3 Solve the resulting equation.

Step 4 Check your solution in the original equation to make sure that you have not obtained any extraneous roots.

Example 1: Solve $2\sqrt{4x + 8} - 4 = 8$.

$$2\sqrt{4x + 8} - 4 = 8$$

Original equation

$$2\sqrt{4x + 8} = 12$$

Add 4 to each side.

$$\sqrt{4x + 8} = 6$$

Isolate the radical.

$$4x + 8 = 36$$

Square each side.

$$4x = 28$$

Subtract 8 from each side.

$$x = 7$$

Divide each side by 4.

Check

$$2\sqrt{4(7) + 8} - 4 \stackrel{?}{=} 8$$

$$2\sqrt{36} - 4 \stackrel{?}{=} 8$$

$$2(6) - 4 \stackrel{?}{=} 8$$

$$8 = 8$$

The solution $x = 7$ checks.

Example 2: Solve $\sqrt{3x + 1} = \sqrt{5x} - 1$.

$$\sqrt{3x + 1} = \sqrt{5x} - 1$$

Original equation

$$3x + 1 = 5x - 2\sqrt{5x} + 1$$

Square each side.

$$2\sqrt{5x} = 2x$$

Simplify.

$$\sqrt{5x} = x$$

Isolate the radical.

$$5x = x^2$$

Square each side.

$$x^2 - 5x = 0$$

Subtract $5x$ from each side.

$$x(x - 5) = 0$$

Factor.

$$x = 0 \text{ or } x = 5$$

Check

$$\sqrt{3(0) + 1} = 1, \text{ but } \sqrt{5(0)} - 1 = -1, \text{ so } 0 \text{ is not a solution.}$$

$$\sqrt{3(5) + 1} = 4, \text{ and } \sqrt{5(5)} - 1 = 4, \text{ so the solution is } x = 5.$$

Exercises

Solve each equation.

1. $3 + 2x\sqrt{3} = 5$

2. $2\sqrt{3x + 4} + 1 = 15$

3. $8 + \sqrt{x + 1} = 2$

4. $\sqrt{5 - x} - 4 = 6$

5. $12 + \sqrt{2x - 1} = 4$

6. $\sqrt{12 - x} = 0$

7. $\sqrt{21} - \sqrt{5x - 4} = 0$

8. $10 - \sqrt{2x} = 5$

9. $\sqrt{4 + 7x} = \sqrt{7x - 9}$

10. $4\sqrt[3]{2x + 11} - 2 = 10$

11. $2\sqrt{x - 11} = \sqrt{x + 4}$

12. $(9x - 11)^{\frac{1}{2}} = x + 1$