

Aim: How do we choose an appropriate method for solving quadratic equations?

Lesson

Method for solving quadratic equations:

First, transform a quadratic equation into standard form, and then decide which method to use.

1. Solve quadratic equations by factoring

Example:

$$x^2 + 5x + 6 = 0$$

$$(x + 3)(x + 2) = 0$$

$$x + 3 = 0$$

or $x + 2 = 0$

$$x = -2$$

or $x = -3$

Factoring

Apply zero product property

Solve two first degree equations

Exercise:

a) $x^2 + 7x + 12 = 0$

$$(x + 3)(x + 4) = 0$$

$$\begin{array}{r} x + 3 = 0 \\ -3 \quad -3 \\ \hline x = -3 \end{array}$$

$$\begin{array}{r} x + 4 = 0 \\ -4 \quad -4 \\ \hline x = -4 \end{array}$$

c) $x^2 - 16x + 63 = 0$

$$(x - 9)(x - 7) = 0$$

$$\begin{array}{r} x - 9 = 0 \\ +9 \quad +9 \\ \hline x = 9 \end{array}$$

$$\begin{array}{r} x - 7 = 0 \\ +7 \quad +7 \\ \hline x = 7 \end{array}$$

b) $x^2 + x - 20 = 0$

$$(x + 5)(x - 4) = 0$$

$$\begin{array}{r} x + 5 = 0 \\ -5 \quad -5 \\ \hline x = -5 \end{array}$$

$$\begin{array}{r} x - 4 = 0 \\ +4 \quad +4 \\ \hline x = 4 \end{array}$$

d) $2x^2 + x - 15 = 0$

$$(2x - 5)(x + 3) = 0$$

$$\begin{array}{r} 2x - 5 = 0 \\ +5 \quad +5 \\ \hline \frac{2x}{2} = \frac{5}{2} \\ x = 2.5 \end{array}$$

$$\begin{array}{r} x + 3 = 0 \\ -3 \quad -3 \\ \hline x = -3 \end{array}$$

~~$$\begin{array}{r} -30x^2 \\ -\frac{5x}{2} \\ \hline 1x \\ \frac{6x}{2} \end{array}$$~~

2. Solve quadratic equations by factoring special cases

Example:

$$x^2 - 9 = 0$$

$$(x + 3)(x - 3) = 0$$

$$x + 3 = 0$$

or $x - 3 = 0$

$$x = -3$$

or $x = 3$

Factoring, since $A^2 - B^2 = (A + B)(A - B)$

Apply zero product property

Solve two first degree equations

Exercise:

a) $4x^2 - 25 = 0$

$$(2x + 5)(2x - 5) = 0$$

$$\begin{array}{r} 2x + 5 = 0 \\ -5 \quad -5 \\ \hline \frac{2x}{2} = \frac{-5}{2} \\ x = -2.5 \end{array}$$

$$\begin{array}{r} 2x - 5 = 0 \\ +5 \quad +5 \\ \hline \frac{2x}{2} = \frac{5}{2} \\ x = 2.5 \end{array}$$

b) $x^2 + 8x + 16 = 0$

$$(x + 4)(x + 4) = 0$$

$$(x + 4)^2 = 0$$

$$\begin{array}{r} x + 4 = 0 \\ -4 \quad -4 \\ \hline x = -4 \end{array}$$

3. Solve quadratic equations using quadratic formula

If $ax^2 + bx + c = 0$ and $a \neq 0$, then $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

Example:

$$x^2 + 5x + 6 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-5 \pm \sqrt{5^2 - (4)(1)(6)}}{2(1)}$$

Use the quadratic equation

Substitute 1 for a, 5 for b, and 6 for c

$$x = \frac{-5 \pm \sqrt{25 - 24}}{2}$$

Simplify

$$x = \frac{-5 \pm 1}{2}$$

Simplify

$$x = \frac{-5 + 1}{2}$$

or $x = \frac{-5 - 1}{2}$

Calculate two solutions

$$x = -2$$

or $x = -3$

Write two solutions

The solutions are -2 and -3

Exercise: Solving the following quadratic equations using quadratic formula.

a) $x^2 + 7x + 12 = 0$ $a=1$ $b=7$ $c=12$

$$x = \frac{-7 \pm \sqrt{7^2 - 4(1)(12)}}{2(1)} = \frac{-7 \pm \sqrt{25}}{2}$$

$$x = \frac{-7 \pm 5}{2} = \boxed{-1, -6}$$

d) $x^2 + 4x + 2 = 0$ $a=1$ $b=4$ $c=2$

$$x = \frac{-4 \pm \sqrt{4^2 - 4(1)(2)}}{2(1)} = \frac{-4 \pm \sqrt{8}}{2}$$

$$x = \frac{-4 \pm 2\sqrt{2}}{2} = \boxed{-2 \pm \sqrt{2}}$$

b) $x^2 + 8x + 16 = 0$ $a=1$ $b=8$ $c=16$

$$x = \frac{-8 \pm \sqrt{8^2 - 4(1)(16)}}{2(1)} = \frac{-8 \pm \sqrt{0}}{2}$$

$$x = \frac{-8}{2} = \boxed{-4}$$

e) $x^2 + 5x + 3 = 0$ $a=1$ $b=5$ $c=3$

$$x = \frac{-5 \pm \sqrt{5^2 - 4(1)(3)}}{2(1)} = \frac{-5 \pm \sqrt{13}}{2}$$

$$x = \frac{-5 \pm \sqrt{13}}{2}$$

c) $4x^2 - 25 = 0$ $a=4$ $b=0$ $c=-25$

$$x = \frac{-0 \pm \sqrt{0^2 - 4(4)(-25)}}{2(4)} = \frac{\pm \sqrt{400}}{8}$$

$$x = \frac{\pm 20}{8} = \boxed{\pm 2.5}$$

f) $12x^2 + x - 35 = 0$ $a=12$ $b=1$ $c=-35$

$$x = \frac{-1 \pm \sqrt{1^2 - 4(12)(-35)}}{2(12)} = \frac{-1 \pm \sqrt{1681}}{24}$$

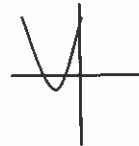
$$x = \frac{-1 \pm 41}{24} = \boxed{-1.75, 5/3}$$

4. Solve quadratic equations by graphing

Example:

$$x^2 + 5x + 6 = 0$$

The x-intercepts are -2 and -3



5. Exercise: Solve the following equations by appropriate method.

a) $x^2 - 5x + 4 = 0$

$$x = \boxed{1, 4}$$

c) $x^2 + 3x + 1 = 0$ $a=1$ $b=3$ $c=1$

$$x = \frac{-3 \pm \sqrt{3^2 - 4(1)(1)}}{2(1)} = \frac{-3 \pm \sqrt{5}}{2}$$

b) $9x^2 + 24x + 16 = 0$

$$(3x + 4)^2 = 0$$

$$3x + 4 = 0$$

$$\underline{4 - 4}$$

$$3x = -4$$

$$x = \boxed{-4/3}$$

d) $25x^2 - 36 = 0$

$$(5x + 6)(5x - 6) = 0$$

$$\frac{5x + 6 = 0}{-6 - 6}$$

$$5x = -6$$

$$x = \boxed{-6/5}$$

$$\frac{5x - 6 = 0}{+6 + 6}$$

$$5x = 6$$

$$x = \boxed{6/5}$$