KEY - MILLONZI

UNIT 6 – EXPONENTIAL FUNCTIONS – 2016-17 (Millonzi)
Linear vs. Exponential Functions (Day 1)

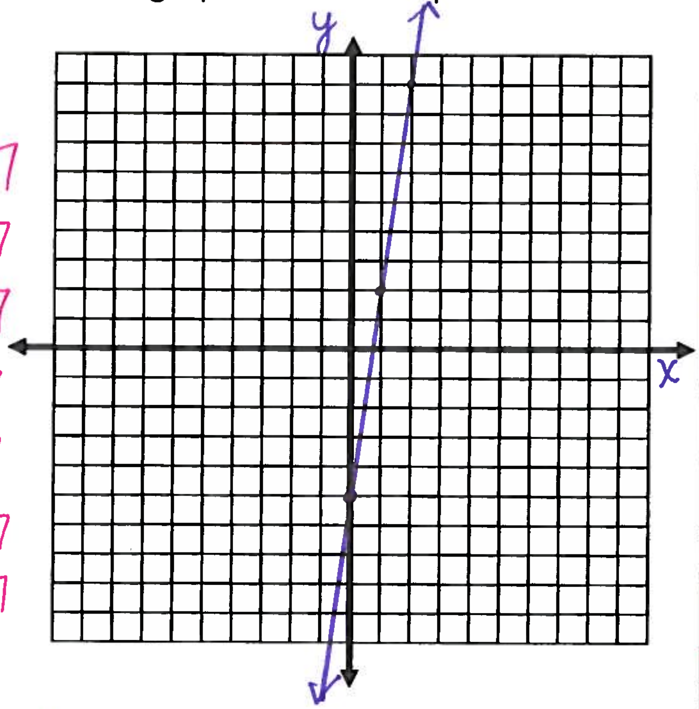
Complete the tables below; graph each set of points.

1.

x (x)	f(x) (y)
0	-5
1	2
2	9
3	16
4	23
5	30
6	37
7	44

+7 +7 +7 +7 +7 +7 +7

"Common Difference"



Key Components

Pattern of Table: Add 7 to each f(x).

Type of Function: Linear

Rate of Change: (ROC) (SLOPE) $\boxed{\frac{7}{1}}$

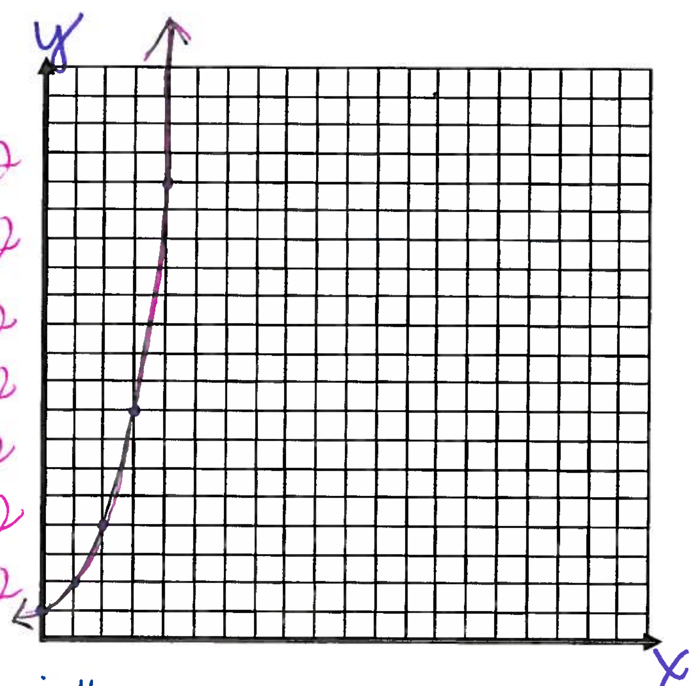
Same throughout graph

2.

x (x)	f(x) (y)
0	1
1	2
2	4
3	8
4	16
5	32
6	64
7	128

·2 ·2 ·2 ·2 ·2 ·2 ·2

"Common Ratio"



Key Components

Pattern of Table: Multiply by 2

Type of Function: Exponential

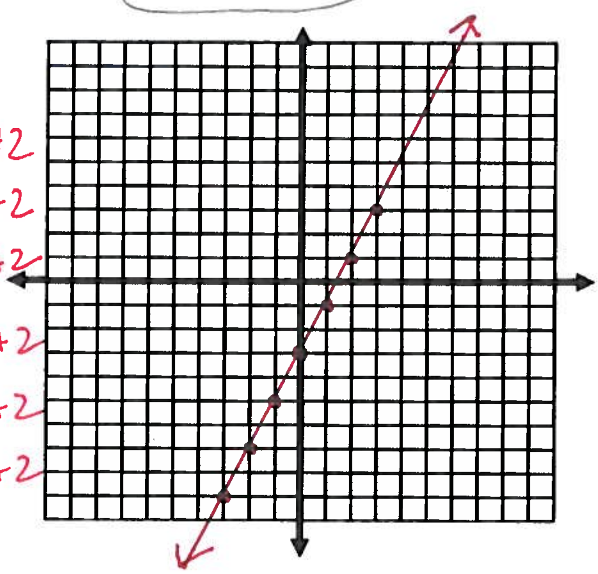
Rate of Change: (ROC) Different for every interval. Between x=0 and x=1, ROC is $\frac{1}{1}$. Between x=2 and x=3, ROC is $\frac{4}{1}$.

Linear Functions	Table Pattern shows Add or Subtract by same number: **This pattern is called a** Common Difference, d Rate of Change is Constant (Same) between intervals
Exponential Functions	Table Pattern shows Multiply or Divide by same number: **This pattern is called a** Common Ratio, r Rate of Change is Different between intervals

Use your Calculator to fill in the Tables.

3. Use the function g(x) = 2x – 3 to fill in the table below and graph.

x	g(x)	
-3	-9	+2
-2	-7	+2
-1	-5	+2
0	-3	+2
1	-1	+2
2	1	+2
3	3	



a) What type of function is this and why? Linear because there is a common Difference of 2

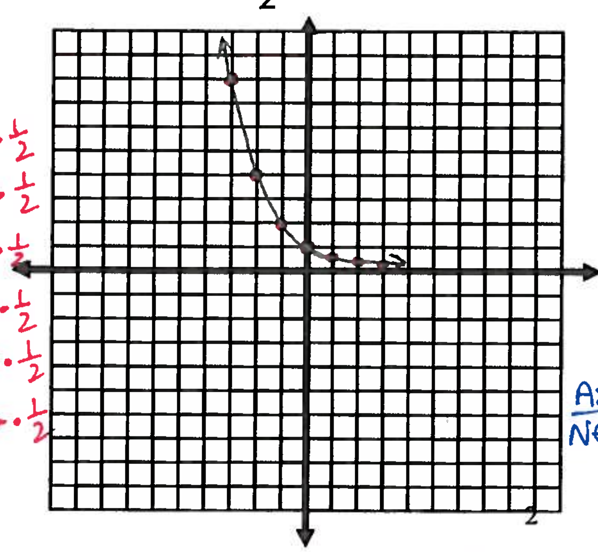
b) What is the domain? $(-\infty, \infty)$

c) What is the range? $(-\infty, \infty)$

d) What is the rate of change? $\frac{2}{1}$ (Constant)

4. Use the function $g(x) = (\frac{1}{2})^x$ to fill in the table below and graph.

x	g(x)	
-3	8	$\cdot\frac{1}{2}$
-2	4	$\cdot\frac{1}{2}$
-1	2	$\cdot\frac{1}{2}$
0	1	$\cdot\frac{1}{2}$
1	.5	$\cdot\frac{1}{2}$
2	.25	$\cdot\frac{1}{2}$
3	.125	



a) What type of function is this and why? Exponential because it has a common Ratio of $\frac{1}{2}$.

b) What is the domain? $(-\infty, \infty)$

c) What is the range? $(0, \infty)$

Asymptotic: Never goes below x-axis

d) What is the rate of change?

Different for every interval.

Ex: Bet. x=-3 and x=-2, R.O.C = $\frac{-4}{1}$

Bet. x=-1 and x=0, R.O.C = -1

UNIT 6 KEY - MICCONZI

IDENTIFYING TYPES OF FUNCTIONS (DAY 2)

Recall Types of Functions and their key components:

Linear functions have a Common Difference with a Constant rate of change.

Exponential functions have a Common Ratio with a Different rate of change.

★ %'s imply Multiplication → Exponential ★

1. After graduation, you are offered two jobs. Cedar Grove Associates offered to start you at $30,000 with a 6% increase every year. Maple Grove Associates offered to start you at $40,000 with a $1200 raise per year. Compare the two jobs offered by completing the table below. Answer the following questions:

either mult. by .06 then add to balance

or Mult. by 1.06 ← Short way

Year	Cedar Grove	Maple Grove
1	$30,000	$40,000
2	30,000 × .06 = 1800 30,000 + 1800 = $31,800	41,200
3	or 31,800 × 1.06 = 33,708	42,400
4	35,730	43,600
5	37,874	44,800
6	40,147	46,000
7	42,556	47,200
8	45,109	48,400
9	47,815	49,600
10	50,684	50,800
11	53,725	52,000
12	56,949	53,200
13	60,366	54,400
14	63,988	55,600

×1.06 ×1.06 ×1.06 ×1.06

+1200 +1200 +1200

a) Cedar Grove models what type of function? Explain. Exponential b/c the same # is multiplied each time

It has a Common Ratio of 1.06

b) Maple Grove models what type of function? Explain. Linear Same # is added each time.

It has a Common Difference of 1200

c) If you plan on moving to a different state in 5 years which company would be the better option for you to choose? Explain. Maple Grove b/c they would pay $44,800 vs. Cedar Grove with $37,874.

d) If your plans change and you don't move, which company would be the better option to choose as a long term career? Explain. Cedar Grove b/c they would pay $63,988 in 14 yrs. vs. $55,600 with Maple Grove.

2. What type of model would be used for each situation below (<u>linear</u>, <u>exponential</u>, or <u>neither</u>)? Explain your reasoning.

a. A savings account that starts with $5000 and receives a deposit of $825 per month.

Linear b/c → Add same amount

b. The value of a house that starts at $150,000 and increases by 1.5% per year.

% means multiply ∴ Exponential

c. Tina owns 4 rabbits. She expects them to double each year.

Means multiply by 2 ∴ Exponential

d. The cost of operating Jelly's Doughnuts is $1600 per week plus $.10 to make each doughnut. $y = 1600 + .10x$ LINEAR

e. The value of John's car that depreciates 20% per year.

% → Multiply by .80 ∴ Exponential

f. The height of a ball that is thrown in the air.

$\boxed{\text{NEITHER}}$ It's actually Quadratic.

3. Which situation could be modeled with an <u>exponential</u> function?

~~(1)~~ the amount of money in Suzy's piggy bank which she adds $10 to each week +
*(2) the amount of money in a certificate of deposit that gets 4% interest each year (×)
~~(3)~~ the amount of money in a savings account where $150 is deducted every month −
~~(4)~~ the amount of money in Jaclyn's wallet which increases and decreases by a different amount each week + or −

4. Which statement below is true about <u>linear</u> functions? "Factor" means multiplic.

~~(1)~~ Linear functions grow by equal factors over equal intervals
*(2) Linear functions grow by equal differences over equal intervals (Constant R.O.C)
(3) Linear functions grown by equal differences over unequal intervals
~~(4)~~ Linear functions grow by unequal factors over equal intervals

5. Given the tables below, classify each as a linear model, exponential model, or neither.

HOURS	MONEY	
1	100	
2	200	×2
3	400	×2
4	800	×2
5	1600	×2

Exponential

HOURS	MONEY	
1	100	
2	200	+100
3	300	+100
4	200	−100
5	100	−100

Neither

HOURS	MONEY	
1	100	
2	250	+150
3	400	+150
4	550	+150
5	700	+150

Linear

Key Millonzi

EXPLORING EXPONENTIAL FUNCTION
GROWTH & DECAY (DAY 3)

Linear Functions		Exponential Functions	
General Equation $y = ax + b$ $y = mx + b$	**Function Notation** $f(x) = ax + b$	**General Equation** $y = ab^x$	**Function Notation** $f(x) = ab^x$
		(recall: variable is the exponent for an exponential function)	
a = slope **b =** y-int.		**a =** initial value (Starting amount) **b =** Rate of growth or decay **x =** Time Period (ex: # years)	

Exponential Functions are able to have both a Positive or Negative rate of change

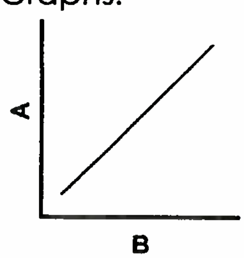
- Positive Rate of Change is called an Exponential Growth

  To Get b# R.O.C: $(1 + r)$

- Negative Rate of Change is called an Exponential Decay

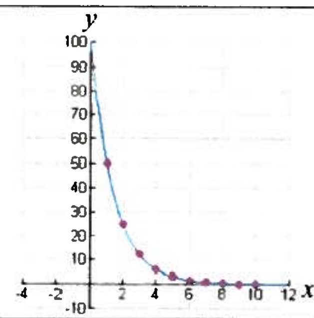
  To Get b# R.O.C: $(1 - r)$

Graphs:



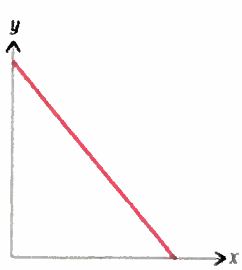
This is a graph of a positively sloped line

Increases from left to right



This is a graph for an exponential Decay

Decreases from left to right

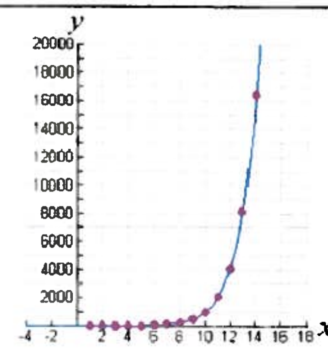


This is a graph of a negatively sloped line

Decreases from left to right

This is a graph for an exponential Growth

Increases from left to right



Exponential Growth & Decay

> Any quantity that grows or decays by a fixed percent at regular intervals is said to possess **exponential growth** or **exponential decay.**

Exponential Function General Equation:

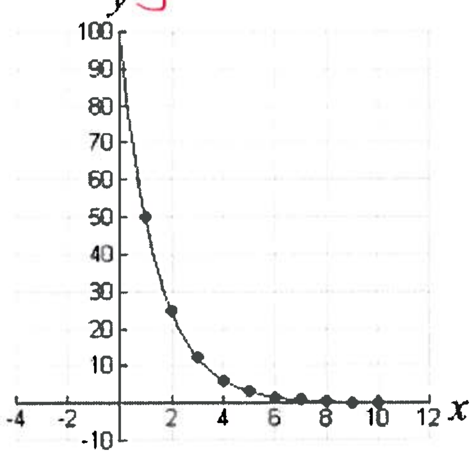
$$y = ab^x$$

Time Period
ex: #years

Starting amount
or Initial Value

Rate: Change % To decimal
Growth: (1 + decimal) $b > 1$
Decay: (1 − decimal) $b < 1$

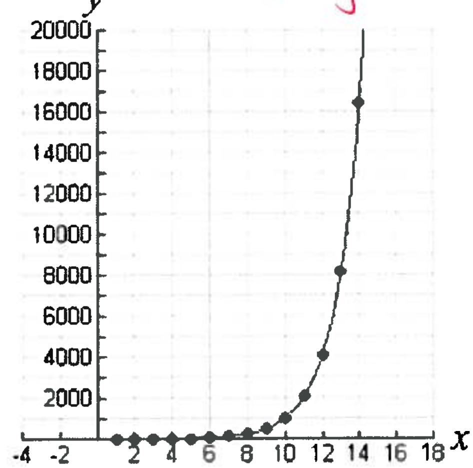
Decreasing from Left to Right



This situation is called an exponential Decay.

$b < 1$ (decimal)

Increasing from Left to Ri



This situation is called an exponential Growth.

$b > 1$

1. Identify the following equations as either linear or exponential:

a) $f(x) = 3^x + 2$ Exponential

b) $2y = -5x + 1$ Linear

c) $y = 7$ HOY Linear

d) $C(x) = 16,332(1.052)^x$ Exponential

For questions #2 – 5:

a. State whether the function is a <u>growth</u> or <u>decay</u>.
b. State the initial value.
C. State the rate of growth or decay

2. $c(t) = 100(.75)^t$

Initial value; $b<1$ Decay

$1-.75 = .25$ 25%

3. $p(n) = 40(1.80)^n$

Initial value; $b>1$ Growth

$1.80-1 = .8$ 80%

4. $t(x) = 10,000(1.02)^x$

Initial Value; $b>1$ Growth

$1.02-1 = .02$ 2%

5. $f(n) = 50(\frac{1}{2})^n$ .5

Initial value; $b<1$ Decay

$1-.5 = .5$ 50%

Steps for writing and solving Functions:

- Identify the type of function (Linear: $y = ax + b$ or Exponential: $y = ab^x$)
- If Exponential:
  - Determine **a#** - initial amount (starting #)
  - Determine **b#**:
    - Exponential Growth: (1+ rate)(%) As decimal
    - Exponential Decay: (1 – rate)(%)

6. A tennis tournament has 128 competitors. Start Half of the competitors are eliminated Decay 50% each round. Write a function to represent the number of competitors that will be left after "x" rounds. Then determine how many players will be left after 5 rounds.

$y = ab^x$

$y = 128(1-.50)^x$

$x=5$

$y = 128(1-.50)^5$ .5

$y = 4$ 4 players.

7. A three-bedroom house in Burbville was purchased for $190,000. Start If housing prices are expected to increase by 1.8% annually in that town, write a function that models the price of the house in *t* years. Find the price of the house in 6 years

Growth 1.8% .018

$y = ab^t$

$y = 190,000(1+.018)^t$

$y = 190,000(1.018)^6$

$y = \$211,465.86$

8. Jonathan makes a weekly allowance of $25. He also makes $9.50 an hour at his job. Write a function for the amount of money he makes each week based on the amount of hours, h, he works. How much will he make if he works 25 hours?

$y = 25 + 9.50h$

$y = 25 + 9.50(25)$

$y = \$262.50$

LINEAR

Unit 6 – Exponential Functions

HOW TO CHOOSE WHAT FORMULA TO USE

FUNCTION	**Words that indicate what Function to USE**	**FORMULA to USE**
EXPONENTIAL $y = ab^x$ a# initial amount Growth b#: (1+r) Decay b#: (1- r) x= time period (# of years)	% increase Appreciating Growing	Exponential **Growth**: $y = a(1+r)^x$
	% decrease Decaying Depreciating	Exponential **Decay**: $y = a(1-r)^x$
Simple Interest	Simple Interest Rate Simple %	**I(t)=Prt** P = Principal \$ r = % rate as a decimal t = time (years) I(t) = amount of interest earned/owed over time
Compound Interest	Compounded annually Compounded yearly	$A(t) = P(1+r)^t$ P = Principal \$ r = % rate as a decimal t = time (years) A(t) = amount of money earned/owed over time

Kay - Millanzi 2014-5 NOTES

EXPONENTIAL APPLICATION WORD PROBLEMS (DAY 4)

1. In 1995, there were 85 rabbits in Central Park. The population increased by 12% each year. How many rabbits were in Central Park in 2005?

Growth Exponential

us

$y = ab^x$ $y = 85(1+.12)^{10}$ $2005 - 1995 = 10$ yrs. $\boxed{263 \text{ Rabbits}}$

$y = 263.9970977 \rightarrow$ Round down for Living, Breathing Things

2. There are 500 rabbits in Lancaster on February 1st. If the amount of rabbits triples every month, write a function that represents the number of rabbits in Lancaster after "m" months. How many rabbits are there in Lancaster on August 1st? (6 months)

us

Growth Exponential

$f(m) = ab^m$ $\boxed{f(m) = 500(3)^m}$

$f(6) = 500(3)^6$ NOT a %: JUST a factor.

$= \boxed{364{,}500}$ Rabbits

3. The value of an early American coin increases in value at the rate of 6.5% annually. If the purchase price of the coin this year is $1,950, what is the value to the nearest dollar in 15 years?

Growth Exponential

them

$y = ab^x$

$y = 1950(1+.065)^{15}$ $\boxed{\$5015}$

$y = 5015.089963$

4. In 1985, there were 285 cell phone subscribers in the small town of Centerville. The number of subscribers increased by 75% per year after 1985. How many cell phone subscribers were in Centerville in 1994? Growth; Exponential

$y = ab^x$ $1994 - 1985 = 9$ yrs.

$y = 285(1+.75)^9$

$y = 43{,}871.98637 \rightarrow \boxed{43{,}871}$ Round Down for Living Breathing Things

5. The cost of Bob's house in 2005 was $220,000. If his house appreciates in value at a rate of 3.5% every year, what will the price of his house be in 2015? Growth

Exponential

$y = 220{,}000(1+.035)^{10}$ $2015 - 2005 = 10$ yrs.

$y = ab^x$ $y = 310{,}331.7273 = \boxed{\$310{,}332}$

6. Kelli's mom takes a 400 mg dose of aspirin. Each hour, the amount of aspirin in a person's system decreases by about 29%. To the nearest tenth of a milligram, how much aspirin is left in her system after 6 hours?

Them #hours DECAY EXPONENTIAL

$y = ab^x$

$y = 400(1-.29)^6 \rightarrow y = 51.2401357$

$\boxed{51.2 \text{ mg}}$

7. Ryan bought a new computer for \$2,100. The value of the computer decreases by 50% each year. After what year will the value drop below \$300? Decay

us Exponential

$y = ab^x$

$y = 2100(1-.5)^x$ unknown # yrs

below 300

Use TRIAL & Error (you plug #s in for x)

You must show 2 wrong and 1 right trial to get full credit.

$y = 2100(1-.5)^1 = \$1050$

$y = 2100(1-.5)^2 = 525$

$y = 2100(1-.5)^3 = 262.50$

ANSWER: After 3 yrs.

8. Malik bought a new car for \$15,000. His best friend, Will, told him that the car's value will drop by 15% every year. What will the car's value be after 5 years, according to Will? Decay ; Exponential

$y = ab^x$ $y = 15000(1-.15)^5$ $y = 6655.579688$

$\boxed{y \approx \$6656}$

9. In the 2000-2001 school year, the average cost for one year at a four-year college was\$16, 332, which was an increase of 5.2% from the previous year. If this trend were to continue, the equation C(x) = 16, 332(1.052)ˣ could be used to model the cost, C(x), of a college education x years from 2000. Exp. Growth

us/them

a) Find C(4). What does this number represent? The cost of college in 2004.

$C(4) = 16{,}332(1.052)^4 = 20{,}003.33142 \approx \boxed{\$20{,}003}$

b) If this trend continues, how much would parents expect to pay for their new born baby's first year of college? (Assume the child would enter college in 18 years.)

$C(18) = 16{,}332(1.052)^{18} = \boxed{\$40{,}674.54}$

10. In 1993, the population of New Zealand was 3,424,000, with an average annual growth rate of 1.3%. Suppose that this growth rate were to continue.

Exp. Growth

$y = ab^x$

$P(n) = ab^n$, $b = (1+r)$

a) Express the population P as a function of n, the number of years after 1993.

$\boxed{P(n) = 3{,}424{,}000(1+.013)^n}$

b) Estimate New Zealand's population in the year 2010.

2010 − 1993 = 17 yrs.

$P(17) = 3424000(1+.013)^{17}$

$= \boxed{4{,}264{,}757 \text{ people}}$

COMPOUND INTEREST (DAY 5)

In history, banks calculated interest at the end of each year on the original amount either borrowed or invested (the principal). This type of interest was called **simple interest.**

As time went on, banks realized that there was another way to compute interest and make even more money. They called this type of interest **Compound Interest**.

To Compute Compound Interest: Banks calculate the interest for the first period, add it to the Principal, then calculate the interest on the new total for the next period, and so on.

Compound Interest: $A(t) = P(1 + r)^t$

- Where $A(t)$ is the amount earned/owed after t years,
- P is the principal amount (the amount originally borrowed or invested)
- r is the interest rate in decimal form.

Ex 1: Sara had invested $800 in a savings account that paid 4.2% interest compounded annually. How much money was in the account after 4 years, if she left the money untouched?

$A(t)=P(1+r)^t$

$P=800$
$r=.042$
$t=4$

$A(4)=800(1+.042)^4$
$= \$943.11$

Ex 2: If Bailey invested money 4 years ago with an annual interest rate of 3.275% compounded annually, and it is valued at $11,260, how much money did she initially invest?

$A(t)=P(1+r)^t$

$11,260 = P(1+.03275)^4$

unknown

PLUG in
even #5

$11,260 = P\underbrace{(1+.03275)^4}$

$\frac{11,260}{1.137577031} = \frac{P(\cancel{1.137577031})}{\cancel{1.137577031}}$

$9898.230794 = P$

$\boxed{P \approx \$9898.23}$

Ex 3: The savings in a bank account can be modeled by the function $S(t) = 250(1.045)^t$. Which of following, then, is true?

Initial

$.045$
4.5%

(1) The initial amount deposited was $250 and the interest earned was 45%.
(2) The initial amount deposited was $2.50 and the interest earned was 4.5%.
(3) The initial amount deposited was $250 and the interest earned was 4.5%.
(4) The initial amount deposited was $2.50 and the interest earned was 45%.

Ex 4: If \$350 is placed in a savings account that earns 3.5% interest applied once a year (compounded), how much would the savings account be worth after 10 years?

$A(t) = P(1+r)^t$

$A(10) = 350(1+.035)^{10}$

$= 493.7095662$

$\approx \boxed{\$493.71}$

Ex 5: Tammy has a balance of \$5,620 in her savings account. She is moving this account to another bank that has advertised an interest rate of 6.5% per year. Which of the following equations would give Tammy's account worth, W, as a function of the number of years, y, it has been gaining interest?

(1) $W = 5320(.65)^y$
(2) $W = 5620(1.065)^y$
(3) $W = 1.065y + 5620$
(4) $W = 1.65y + 5620$

$A(t) = P(1+r)^t$

$A(t) = 5620(1+.065)^t$

$W = 5620(1.065)^y$

INTRODUCTION TO REGRESSIONS (DAY 6)

Regression Equation: An equation of a function (linear, exponential, etc.) that plotted points follow or create (calculator command creates equations).

Correlation Coefficient (r#): This number describes the strength of the equation in regards to the plotted points. A PERFECT fit equation will have r = 1 or r = -1 depending on the slope of the line or growth of an exponential curve.

(These concepts will be expanded upon later in the year; these are just the basics needed for today)

HOW TO USE A CALCULATOR TO CREATE LINEAR AND EXPONENTIAL EQUATIONS

What's needed for...

- **LINEAR Equations: 2** exact points from table, graph, or word problem
- **EXPONENTIAL Equations: 3** exact points from table, graph, or word problem

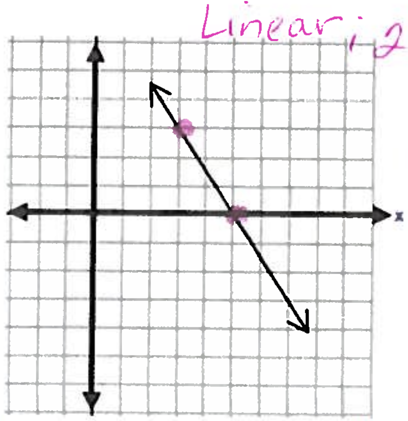
Calculator Steps: (Need to turn Diagnostics On to get r#): $\boxed{2nd}$ $\boxed{\emptyset}$(Catalog) scroll down to Diag. On + Hit Enter twice

- (STAT – EDIT) Put points in a list -- x's go in L_1 and y's go in L_2
- (STAT – CALC) Choose either LinReg (#4) for linear eqn or ExpReg (#0) for exponential eqn
- Verify it is a PERFECT fit equation by looking at the **r#**. MUST = ±1 to be perfect
- Write equation stated by filling in the appropriate values for the coefficients
- **EXPLAIN IN WORDS** the calculator steps you did to get the equation since no math was involved in writing the equations.

Write an equation of the GRAPHED functions below.

- Identify the type of function (Linear or Exponential)
- Choose and write down the correct number of points needed for each function
- Follow the steps above for the calculator

1)



Linear; 2 Points:

(3, 3)
(5, 0)

x	y
3	3
5	0
↑	↑
L_1	L_2

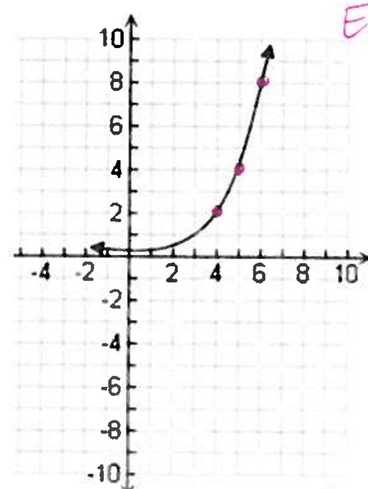
$\boxed{Stat}$ Edit
L_1 : x-values
L_2 : y-values
$\boxed{Stat}$ Calc
#4 : LinReg

$y = ax + b$

$\boxed{y = -1.5x + 7.5}$

r = -1 (Perfect)

2)



Exponential; 3 Points:

(4, 2)
(5, 4)
(6, 8)

x	y
4	2
5	4
6	8
↑	↑
L_1	L_2

$\boxed{Stat}$ Edit
L_1 : x-values
L_2 : y-values
$\boxed{Stat}$ Calc
#∅ : ExpReg

$y = a(b)^x$

$\boxed{y = .125(2)^x}$

r = 1 (Perfect)

3) Write an equation of a line that goes through the points (-2, -11) and (3, 14).

L_1	L_2
-2	-11
3	14

LinReg $\boxed{y=5x-1}$ $r=1$ (perfect)

4) Write the equation of the exponential function that goes through the following points: (-1, 16), (2, 6.75), and (1, 9)

L_1	L_2
-1	16
2	6.75
1	9

ExpReg $\boxed{y=12(.75)^x}$ $r=-1$ (perfect)

5) Each day, Toni records the height of a plant for her science lab. Her data are shown in the table below:

L_1 : X
L_2 : Y

Day (n)	1	2	3	4	5
Height (cm)	3.0	4.5	6.0	7.5	9.0

+1.5 +1.5 +1.5 +1.5 ← Common Difference

The plant continues to grow at a constant daily rate. Write an equation to represent h(n), the height of the plant on the nth day.

Linear (has common difference)

Take 2 points and do LinReg on Calculator.

L_1	L_2
1	3
2	4.5

LinReg $\boxed{y=1.5x+1.5}$ $r=1$ (perfect)

6) Write an exponential function f(x) for the table shown to the right. Only need 3 pts.

L_1
L_2

x	0	1	2	3
f(x)	13	39	117	351

×3 ×3 ×3 ← Common Ratio

$f(x)=a(b)^x$

ExpReg $y=13(3)^x$

$\boxed{f(x)=13(3)^x}$ $r=1$ (perfect)

7) Jackson is starting an exercise program. The first day he will spend 30 minutes on the treadmill. He will increase his time on the treadmill by 2 minutes each day. Write an equation for T(d), the time, in minutes, on the treadmill on day d.

Adding 2 minutes each day is showing a Common Difference, So this is LINEAR. Only need 2 points.

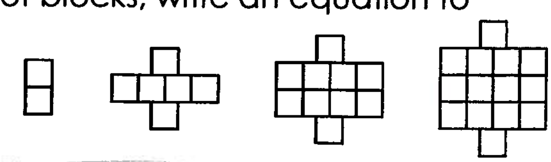
X	Y
1	30
2	32
3	34

LinReg $\boxed{T(d)=2d+28}$ $r=1$ (perfect)

8) Given the illustration shown below of a pattern of blocks, write an equation to represent this model.

LINEAR, so pick 2 points + do LinReg. $\boxed{y=4x-2}$ $r=1$ (perfect)

X	Y
1	2
2	6



Term 1 (1,2) Term 2 (2,6) Term 3 (3,10) Term 4 (4,14)

+4 +4 +4

Com. Diff., So LINEAR