Exploring Rational Functions

Lesson Information
By Jon Schweig
Subject:
Pre-Calculus Algebra
Topic:
Functions Graphing
Technology:
Graphing Calculator
Level:
Moderate
Activity Structure:
Self-guided Problem Solving
Duration of Activity:
Whole Class Period

Overview:

This lesson has students compare and contrast differences between rational functions to understand vertical asymptotes, horizontal asymptotes, and removable/non-removable discontinuities.

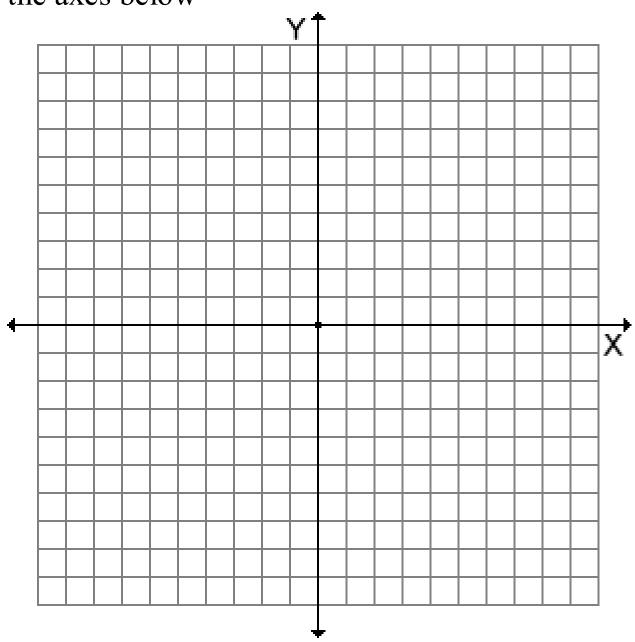
Learning Objectives:

- to see how certain characteristics of the graphs of rational functions can be determined by examining the polynomials in the numerator and denominator.

Materials:

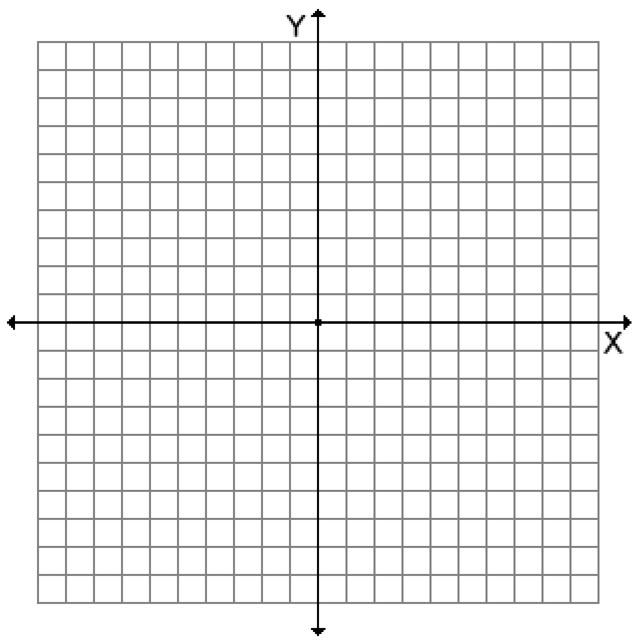
A **rational function** is the quotient of two **polynomials**. The general form of a rational function can be written as $y = p(x)/\,q(x)$, where $p(x)$and $q(x)$are both polynomial functions. In this activity, you will explore the graphs of rational functions.

1. Graph the function $f(x)=\dfrac{x-1}{x}$ on your graphing calculator. What is the **root** of the numerator of the function? What happens to the graph at that x value? Sketch the graph on the axes below



2. Think about the rational function $y = . \dfrac{(x-5)x}{x+1}$. How many roots does the numerator of this function have? What are they?

3. Using your graphing calculator, graph the function shown above. Sketch a graph of that function on the axes below. What happens at the roots of the numerator?



4. What is the *x*-intercept of y=$\dfrac{x+2}{x-3}$? Check you're answer on the graphing calculator.

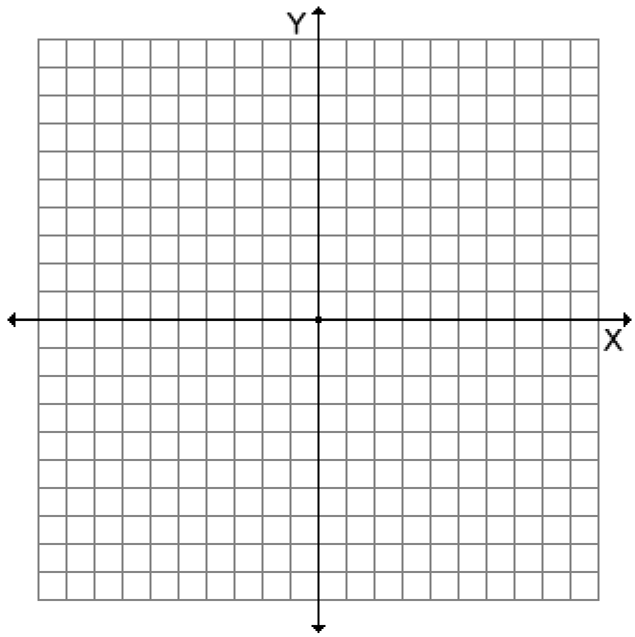
5. Graph y=$\frac{(x+3)(x-3)}{(x-3)}$. What happens at x=3? Why?

6. In general, when will a root of the numerator not be an *x*-intercept on the graph?

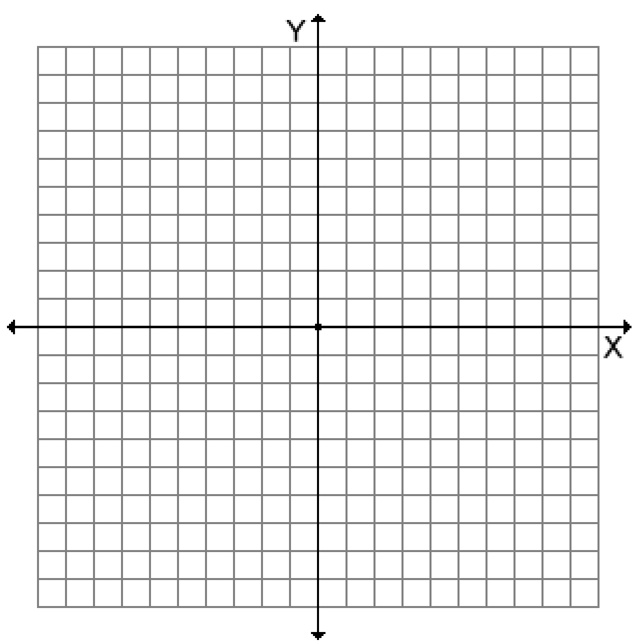
7. What are the *x*-intercepts of y=$\frac{(x+4)(x-3)}{(x-3)}$? Check yourself on your graphing calculator

8. What general statement can you make about the graph of a rational function and the roots of the numerator of the function?

9. In some rational functions, the function *p(x)* in the numerator is a **constant function**, and the function *q(x)* in the denominator is a **linear function**, q(x)=x+B. Consider the function f(x) = 5/(x+2). Graph it on your calculator. Take a few minutes to write down some significant features of this graph. Sketch the graph on the axes below.



10. An **asymptote** is an imaginary line that the graph of a function approaches. Asymptotes are usually shown as dotted lines. Graph the function to be f(x) = 1/x. What appear to be the equations of the asymptotes in the graph of this function?



11. Create three different functions if the form $f(x) = \frac{a}{x+b}$
Write those functions below. What do you notice about the roots of the denominator and the equations of the asymptotes?

12. How are the graphs of the functions $\frac{x}{x-2}$ and $\frac{x(x+1)}{(x+1)(x-2)}$ different? Explain why.

13. For each of the following, write down a function that has the given properties. Then, check your answers by graphing your functions.

a. vertical asymptote at x = -2 ; hole at x = 3

b. vertical asymptotes at x = 4 and x = 0 ; no holes

19. **Horizontal asymptotes** appear on the graphs of some rational functions. The equation of the horizontal asymptote is related to the degree of the numerator and the degree of the denominator.

20. Set up a rational function that has a polynomial of **degree 0** in the numerator, and

a polynomial of **degree 1** in the denominator. Graph that equation. What is the equation of the horizontal asymptote?

21. Make a rational function of degree 1 in the numerator and a polynomial of degree 3 in the denominator. Does the function have a horizontal asymptote? What is it?

22. Set up a rational function with degree 3 in both the numerator and the denominator. Does this function have a horizontal asymptote? What is it?

23. Make a rational function with degree 3 in the numerator and degree 1 in the denominator. Does the function have a horizontal asymptote? What happens in this case?

24. What general rule can you make about when a rational function will have a horizontal asymptote?