Section 3 – Topic 3
Adding and Subtracting Functions

Let $h(x) = 2x^2 + x - 5$ and $g(x) = -3x^2 + 4x + 1$.

Find $h(x) + g(x)$.

$$\mathbf{(2x^2 + x - 5) + (-3x^2 + 4x + 1)}$$
$$\mathbf{= 2x^2 - 3x^2 + x + 4x - 5 + 1}$$
$$\mathbf{= -x^2 + 5x - 4}$$

Find $h(x) - g(x)$.

$$\mathbf{(2x^2 + x - 5) - (-3x^2 + 4x + 1)}$$
$$\mathbf{= 2x^2 + x - 5 + 3x^2 - 4x - 1}$$
$$\mathbf{= 2x^2 + 3x^2 + x - 4x - 5 - 1}$$
$$\mathbf{= 5x^2 - 3x - 6}$$

Let's Practice!

1. Consider the following functions.

$$f(x) = 3x^2 + x + 2$$
$$g(x) = 4x^2 + 2(3x - 4)$$
$$h(x) = 5(x^2 - 1)$$

a. Find $f(x) - g(x)$.

$$(3x^2 + x + 2) - \left(4x^2 + 2(3x - 4)\right)$$
$$(3x^2 + x + 2) - (4x^2 + 6x - 8)$$
$$3x^2 + x + 2 - 4x^2 - 6x + 8$$
$$3x^2 - 4x^2 + x - 6x + 2 + 8$$
$$-x^2 - 5x + 10$$

b. Find $g(x) - h(x)$.

$$4x^2 + 2(3x - 4) - 5(x^2 - 1)$$
$$4x^2 + 6x - 8 - (5x^2 - 5)$$
$$4x^2 + 6x - 8 - 5x^2 + 5$$
$$4x^2 - 5x^2 + 6x - 8 + 5$$
$$-x^2 + 6x - 3$$

Try It!

2. Recall the functions we used earlier.

$$f(x) = 3x^2 + x + 2$$
$$g(x) = 4x^2 + 2(3x - 4)$$
$$h(x) = 5(x^2 - 1)$$

a. Let $m(x)$ be $f(x) + g(x)$. Find $m(x)$.

$$\boldsymbol{m(x) = \left(3x^2 + x + 2\right) + \left(4x^2 + 2(3x - 4)\right)}$$
$$\boldsymbol{m(x) = \left(3x^2 + x + 2\right) + \left(4x^2 + 6x - 8\right)}$$
$$\boldsymbol{m(x) = 3x^2 + x + 2 + 4x^2 + 6x - 8}$$
$$\boldsymbol{m(x) = 3x^2 + 4x^2 + x + 6x + 2 - 8}$$
$$\boldsymbol{m(x) = 7x^2 + 7x - 6}$$

b. Find $h(x) - m(x)$.

$$\boldsymbol{5\left(x^2 - 1\right) - \left(7x^2 + 7x - 6\right)}$$
$$\boldsymbol{5x^2 - 5 - 7x^2 - 7x + 6}$$
$$\boldsymbol{5x^2 - 7x^2 - 7x - 5 + 6}$$
$$\boldsymbol{-2x^2 - 7x + 1}$$

BEAT THE TEST!

1. Consider the functions below.

$$f(x) = 2x^2 + 3x - 5$$
$$g(x) = 5x^2 + 4x - 1$$

Which of the following is the resulting polynomial when $f(x)$ is subtracted from $g(x)$?

Ⓐ $-3x^2 - x - 4$
Ⓑ $-3x^2 + 7x - 6$
Ⓒ $\mathbf{3x^2 + x + 4}$
Ⓓ $3x^2 + 7x - 6$

Answer: C

Algebra Wall

Want some help? You can always ask questions on the Algebra Wall and receive help from other students, teachers, and Study Experts. You can also help others on the Algebra Wall and earn Karma Points for doing so. Go to AlgebraNation.com to learn more and get started!

Section 3 – Topic 4
Multiplying Functions

Use the distributive property and modeling to perform the following function operations.

Let $f(x) = 3x^2 + 4x + 2$ and $g(x) = 2x + 3$.

Find $f(x) \cdot g(x)$.

$(3x^2 + 4x + 2)(2x + 3)$
$3x^2 \cdot 2x + 3x^2 \cdot 3 + 4x \cdot 2x + 4x \cdot 3 + 2 \cdot 2x + 2 \cdot 3$
$6x^3 + 9x^2 + 8x^2 + 12x + 4x + 6$
$6x^3 + 17x^2 + 16x + 6$

	$3x^2$	$4x$	2
$2x$	$6x^3$	$8x^2$	$4x$
3	$9x^2$	$12x$	6

$6x^3 + 17x^2 + 16x + 6$

Let $m(y) = 3y^5 - 2y^2 + 8$ and $p(y) = y^2 - 2$.

Find $m(y) \cdot p(y)$.

	$3y^5$	$0y^4$	$0y^3$	$-2y^2$	$0y$	8
y^2	$3y^7$	0	0	$-2y^4$	0	$8y^2$
$0y$	0	0	0	0	0	0
-2	$-6y^5$	0	0	$4y^2$	0	-16

$$3y^7 - 6y^5 - 2y^4 + 12y^2 - 16$$

Let's Practice!

1. Hutchinson Square is an urban park in downtown Summerville, South Carolina. Suppose the park is hosting a spring carnival and one of the events will be a treasure hunt. The hunt takes place in a large sandbox. The length of the sandbox in inches, $l(x)$, can be represented by the expression $(x + 12)$, and the width of the sandbox in inches, $w(x)$, can be represented by the expression $(x + 4)$.

   a. Find $w(x) \cdot l(x)$.

   $\mathbf{x^2 + 16x + 48}$

   b. Circle the best answer to complete the following statement.

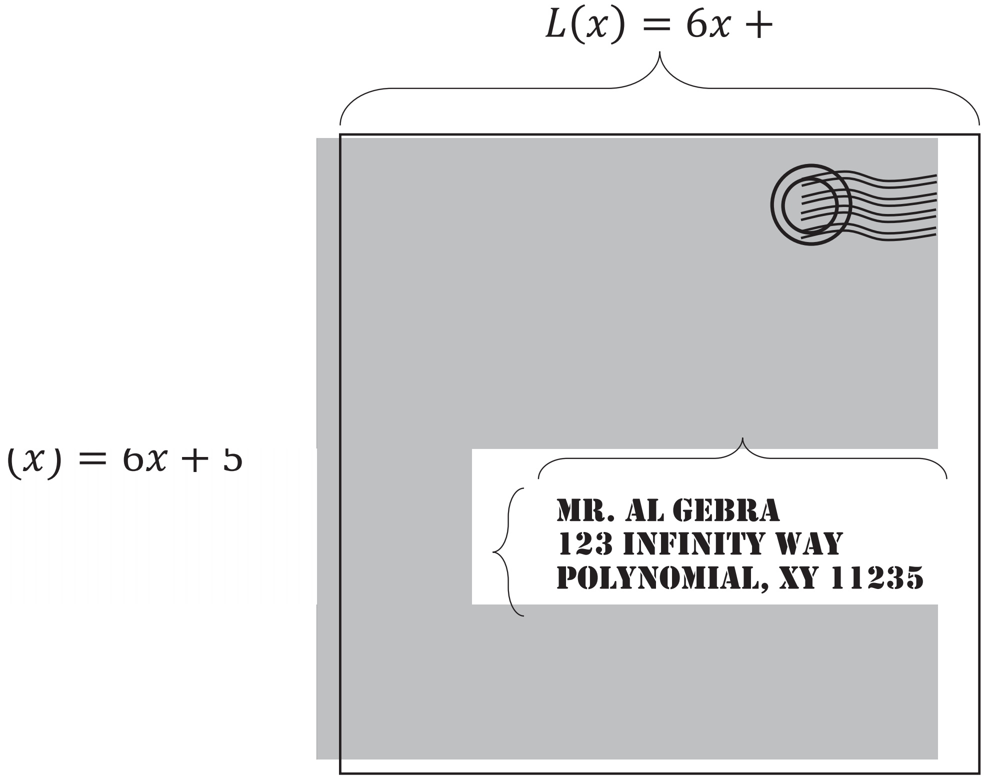
   The product of $w(x)$ and $l(x)$ is equivalent to the area | elevation | perimeter | volume of the sandbox.

   c. The carnival staff need to be sure they purchase enough sand to completely fill the sandbox. If the height of the box in inches, $h(x)$, can be represented by $(x - 28)$, write an expression to represent the volume of the box.

   $\mathbf{x^3 - 12x^2 - 400x - 1344}$

Try It!

2. The envelope below has a mailing label:



a. Let $A(x) = L(x) \cdot W(x) - M(x) \cdot N(x)$. Find $A(x)$.

$\mathbf{L(x) \cdot W(x) = (6x + 5)(6x + 5)}$
$\mathbf{= 6x \cdot 6x + 6x \cdot 5 + 5 \cdot 6x + 5 \cdot 5}$
$\mathbf{= 36x^2 + 30x + 30x + 25}$
$\mathbf{= 36x^2 + 60x + 25}$

$\mathbf{M(x) \cdot N(x) = (x + 4)(x + 2)}$
$\mathbf{= x \cdot x + x \cdot 2 + 4 \cdot x + 4 \cdot 2}$
$\mathbf{= x^2 + 2x + 4x + 12}$
$\mathbf{= x^2 + 6x + 8}$

$\mathbf{A(x) = L(x) \cdot W(x) - M(x) \cdot N(x)}$
$\mathbf{= (36x^2 + 60x + 25) - (x^2 + 6x + 8)}$
$\mathbf{= 36x^2 + 60x + 25 - x^2 - 6x - 8}$
$\mathbf{= 35x^2 + 54x + 17}$

b. What does the function $A(x)$ represent in this problem?

The area of the front of the envelope excluding the address label.

BEAT THE TEST!

1. The length of the sides of a square are s inches long. A rectangle is six inches shorter and eight inches wider than the square.

Part A: Express both the length and the width of the rectangle as a function of a side of the square.

Length: $L(s) = s - 6$
Width: $W(s) = s + 8$

Part B: Write a function to represent the area of the rectangle in terms of the sides of the square.

$A(s) = (s-6)(s+8) = s^2 + 8s - 6s - 48 = s^2 + 2s - 48$

Algebra Wall

Want some help? You can always ask questions on the Algebra Wall and receive help from other students, teachers, and Study Experts. You can also help others on the Algebra Wall and earn Karma Points for doing so. Go to AlgebraNation.com to learn more and get started!

Section 3 – Topic 5
Closure Property

When we add two integers, what type of number is the sum?

Integer

When we multiply two irrational numbers, what type of number is the product?

It could be rational or irrational.

$\sqrt{2} \cdot \sqrt{8} = \sqrt{16} = 4$

$\sqrt{2} \cdot \sqrt{3} = \sqrt{6}$

A set is ___closed___ for a specific operation if and only if the operation on two elements of the set ***always*** produces an element of the same set.

Are integers closed under addition? Justify your answer.

Yes, the sum of integers always results in an integer.

Are irrational numbers closed under multiplication? Justify your answer.

No, the product of irrational numbers is not always irrational.

Let's apply the closure property to polynomials.

Are the following statements true or false? If false, give a counterexample.

Polynomials are closed under addition.
True

Polynomials are closed under subtraction.
True

Polynomials are closed under multiplication.
True

Let's Practice!

1. Check the boxes for the following sets that are closed under the given operations.

Set	$+$	$-$	$\times$
$\{0, 1, 2, 3, 4, \dots\}$	☒	☐	☒
$\{\dots, -4, -3, -2, -1\}$	☒	☐	☐
$\{\dots, -3, -2, -1, 0, 1, 2, 3, \dots\}$	☒	☒	☒
{rational numbers}	☒	☒	☒
{polynomials}	☒	☒	☒

Try It!

2. Ms. Sanabria claims that the closure properties for polynomials are analogous to the closure properties for integers. Mr. Roberts claims that the closure properties for polynomials are analogous to the closure properties of whole numbers. Who is correct? Explain your answer.

Ms. Sanabria is correct because integers and polynomials are both closed under addition, subtraction, and multiplication. Mr. Roberts is not correct because whole numbers are not closed under subtraction [ex: 2 – 5 = -3, and -3 is not a whole number]. Whole numbers are only closed under addition and multiplication, while polynomials are closed under addition, subtraction, and multiplication.

BEAT THE TEST!

1. Choose from the following words and expressions to complete the statement below.

$2x^5 + (3y)^{-2} - 2$	$(5y)^2 + 4x + 3y^3$	$5y^{-1} + 7x^2 + 8y^2$
integers	variables	whole numbers
coefficients	rational numbers	exponents

The product of $5x^4 - 3x^2 + 2$ and $\underline{(5y)^2 + 4x + 3y^3}$ illustrates the closure property because the <u>exponents</u> of the product are <u>whole numbers</u> and the product is a polynomial.

Algebra Wall	Want some help? You can always ask questions on the Algebra Wall and receive help from other students, teachers, and Study Experts. You can also help others on the Algebra Wall and earn Karma Points for doing so. Go to AlgebraNation.com to learn more and get started!