Lesson 11: Completing the Square

Classwork

Opening Exercise

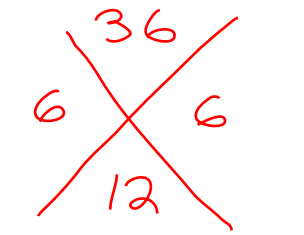
Rewrite the following perfect square quadratic expressions in standard form. Describe patterns in the coefficients for the factored form, $(x + A)^2$, and the standard form, $x^2 + bx + c$.

FACTORED FORM	WRITE THE FACTORS	DISTRIBUTE	STANDARD FORM
Example: $(x + 1)^2$	$(x+1)(x+1)$	$x^2+1x+1x+1$	x^2+2x+1
$(x + 2)^2$	$(x+2)(x+2)$	$x^2+2x+2x+4$	x^2+4x+4
$(x + 3)^2$	$(x+3)(x+3)$	$x^2+3x+3x+9$	x^2+6x+9
$(x + 4)^2$			$x^2+8x+16$
$(x + 5)^2$			$x^2+10x+25$
$(x + 20)^2$			$x^2+40x+400$

Example

Now try working backward. Rewrite the following standard form quadratic expressions as perfect squares.

STANDARD FORM	FACTORED FORM
$x^2 + 12x + 36$	$(x+6)^2$
$x^2 - 12x + 36$	$(x-6)^2$
$x^2 + 20x + 100$	$(x+10)^2$
$x^2 - 3x + \frac{9}{4}$	$(x-\frac{3}{2})^2$
$x^2 + 100x + 2500$	$(x+50)^2$
$x^2 + 8x + 3$	can't solve



$(x+6)(x+6)$

$(x+6)^2$

Exploratory Challenge

Find an expression equivalent to $x^2 + 8x + 3$ that includes a perfect square binomial.

$x^2 + 8x + 16$ -16 $+3$

$(x+4)^2 - 13$ (Vertex Form)

Exercises

$\left(\frac{b}{2}\right)^2$

Rewrite each expression by completing the square.

1. $a^2 - 4a + 15$

$a^2 - 4a + 4$ -4 $+15$

$(a-2)^2 + 11$

2. $n^2 - 2n - 15$

$n^2 - 2x + 1$ -1 -15

$(n-1)^2 - 16$

$\left(\frac{20}{2}\right)^2 = 100$

3. $c^2 + 20c - 40$

$c^2 + 20c + 100 + 100 - 40$

$(c+10)^2 - 140$

$\left(\frac{1000}{2}\right)^2 = 250000$

4. $x^2 - 1000x + 60\,000$

$x^2 - 1000x + 250000$ -250000 $+60,000$

$(x-500)^2 - 190,000$

5. $y^2 - 3y + 10$

6. $k^2 + 7k + 6$

7. $z^2 - 0.2z + 1.5$

8. $p^2 + 0.5p + 0.1$

9. $j^2 - \frac{3}{4}j + \frac{3}{4}$

10. $x^2 - bx + c$

Lesson Summary

Just as factoring a quadratic expression can be useful for solving a quadratic equation, completing the square also provides a form that facilitates solving a quadratic equation.

Problem Set

Rewrite each expression by completing the square.

1. $q^2 + 12q + 32$

2. $m^2 - 4m - 5$

3. $x^2 - 7x + 6.5$

4. $a^2 + 70a + 1225$

5. $z^2 - 0.3z + 0.1$

6. $y^2 - 6by + 20$

7. Which of these expressions would be most easily rewritten by factoring? Justify your answer.